



22117204

**MATHEMATICS
HIGHER LEVEL
PAPER 2**

Thursday 5 May 2011 (morning)

Candidate session number

2 hours

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



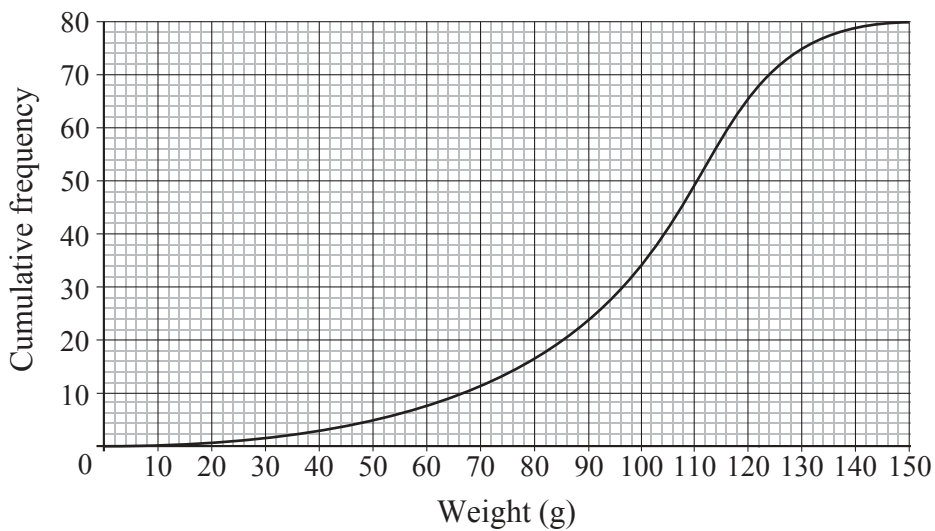
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The cumulative frequency graph below represents the weight in grams of 80 apples picked from a particular tree.



- (a) Estimate the
 - (i) median weight of the apples;
 - (ii) 30th percentile of the weight of the apples. [2 marks]
- (b) Estimate the number of apples which weigh more than 110 grams. [2 marks]

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2. [Maximum mark: 6]

Consider the function $f(x) = x^3 - 3x^2 - 9x + 10$, $x \in \mathbb{R}$.

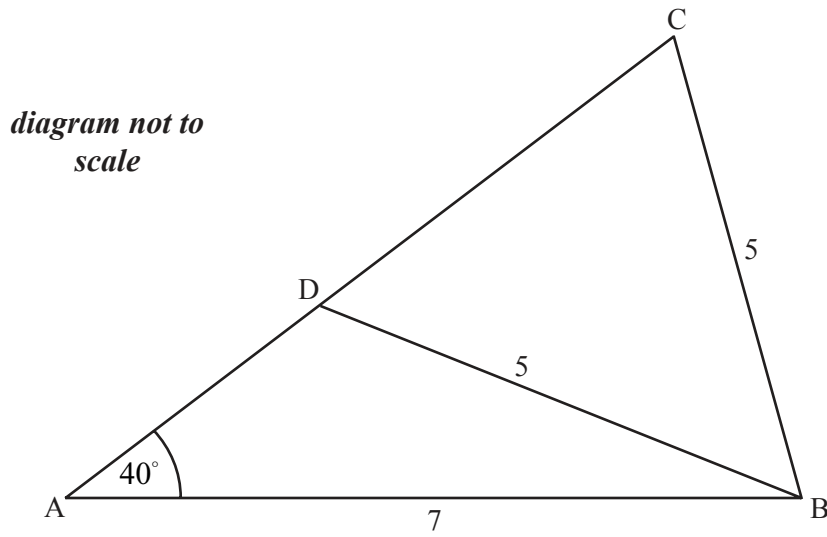
- (a) Find the equation of the straight line passing through the maximum and minimum points of the graph $y = f(x)$. [4 marks]
- (b) Show that the point of inflexion of the graph $y = f(x)$ lies on this straight line. [2 marks]

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3. [Maximum mark: 5]

Given $\triangle ABC$, with lengths shown in the diagram below, find the length of the line segment $[CD]$.



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4. [Maximum mark: 5]

The function $f(x) = 4x^3 + 2ax - 7a$, $a \in \mathbb{R}$, leaves a remainder of -10 when divided by $(x - a)$.

(a) Find the value of a . [3 marks]

(b) Show that for this value of a there is a unique real solution to the equation $f(x) = 0$. [2 marks]

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5. [Maximum mark: 5]

(a) Write down the quadratic expression $2x^2 + x - 3$ as the product of two linear factors. [1 mark]

(b) Hence, or otherwise, find the coefficient of x in the expansion of $(2x^2 + x - 3)^8$. [4 marks]

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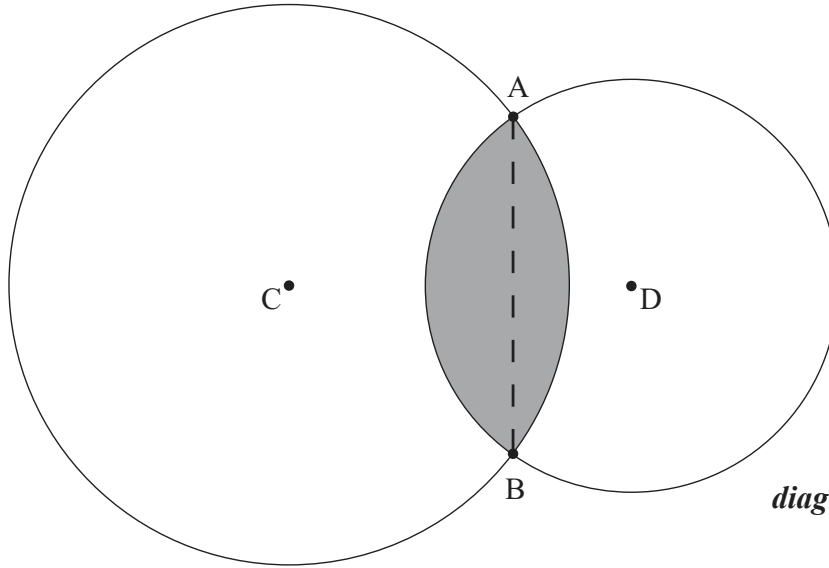
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6. [Maximum mark: 7]

The radius of the circle with centre C is 7 cm and the radius of the circle with centre D is 5 cm. If the length of the chord $[AB]$ is 9 cm, find the area of the shaded region enclosed by the two arcs AB .



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7. [Maximum mark: 7]

A continuous random variable X has a probability density function given by the function $f(x)$, where

$$f(x) = \begin{cases} k(x+2)^2, & -2 \leq x < 0 \\ k, & 0 \leq x \leq \frac{4}{3} \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of k . [2 marks]

(b) Hence find

(i) the mean of X ;

(ii) the median of X . [5 marks]

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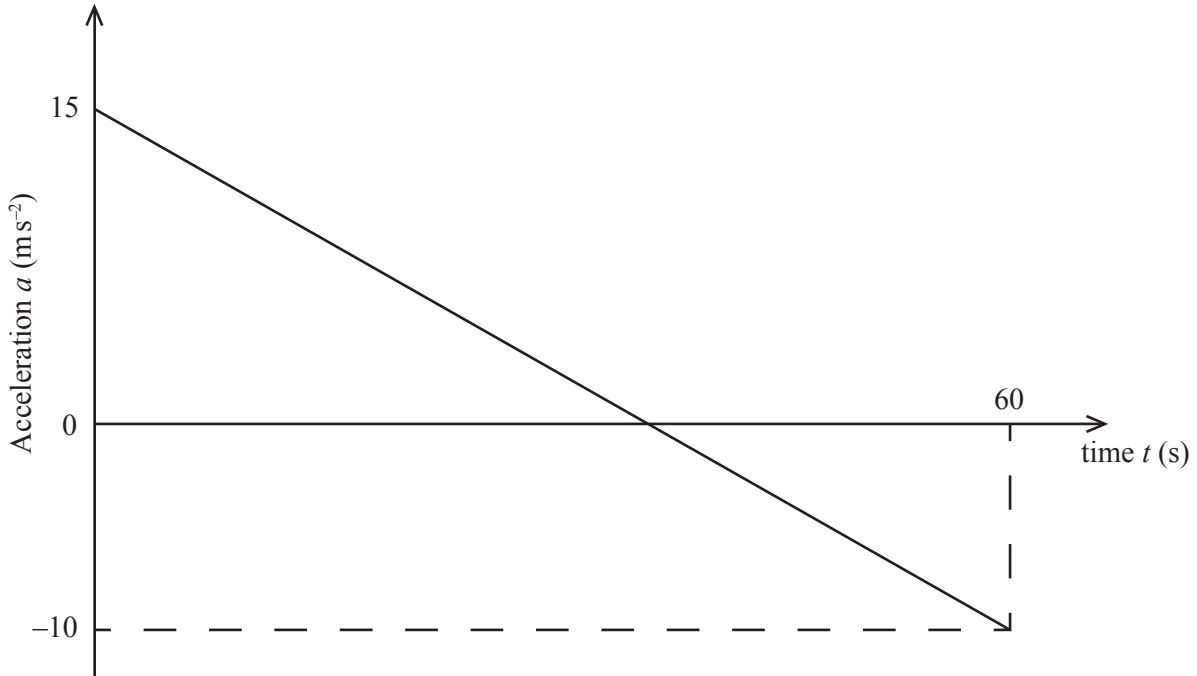
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8. [Maximum mark: 8]

A jet plane travels horizontally along a straight path for one minute, starting at time $t = 0$, where t is measured in seconds. The acceleration, a , measured in ms^{-2} , of the jet plane is given by the straight line graph below.



- (a) Find an expression for the acceleration of the jet plane during this time, in terms of t . [1 mark]

- (b) Given that when $t = 0$ the jet plane is travelling at 125 ms^{-1} , find its maximum velocity in ms^{-1} during the minute that follows. [4 marks]

- (c) Given that the jet plane breaks the sound barrier at 295 ms^{-1} , find out for how long the jet plane is travelling greater than this speed. [3 marks]

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9. [Maximum mark: 6]

Solve the following system of equations.

$$\log_{x+1} y = 2$$

$$\log_{y+1} x = \frac{1}{4}$$

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10. [Maximum mark: 7]

Port A is defined to be the origin of a set of coordinate axes and port B is located at the point (70, 30), where distances are measured in kilometres. A ship S_1 sails from port A at 10:00 in a straight line such that its position t hours after 10:00 is given

by $\mathbf{r} = t \begin{pmatrix} 10 \\ 20 \end{pmatrix}$.

A speedboat S_2 is capable of three times the speed of S_1 and is to meet S_1 by travelling the shortest possible distance. What is the latest time that S_2 can leave port B?

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Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

SECTION B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

11. [Maximum mark: 14]

The equations of three planes, are given by

$$\begin{aligned} ax + 2y + z &= 3 \\ -x + (a+1)y + 3z &= 1 \\ -2x + y + (a+2)z &= k \end{aligned}$$

where $a \in \mathbb{R}$.

- (a) Given that $a = 0$, show that the three planes intersect at a point. [3 marks]

- (b) Find the value of a such that the three planes do **not** meet at a point. [5 marks]

- (c) Given a such that the three planes do **not** meet at a point, find the value of k such that the planes meet in one line and find an equation of this line in the form

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}. \quad [6 \text{ marks}]$$



Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

12. [Maximum mark: 17]

A student arrives at a school X minutes after 08:00, where X may be assumed to be normally distributed. On a particular day it is observed that 40 % of the students arrive before 08:30 and 90 % arrive before 08:55.

- (a) Find the mean and standard deviation of X . [5 marks]
- (b) The school has 1200 students and classes start at 09:00. Estimate the number of students who will be late on that day. [3 marks]
- (c) Maelis had not arrived by 08:30. Find the probability that she arrived late. [2 marks]

At 15:00 it is the end of the school day and it is assumed that the departure of the students from school can be modelled by a Poisson distribution. On average 24 students leave the school every minute.

- (d) Find the probability that at least 700 students leave school before 15:30. [3 marks]
- (e) There are 200 days in a school year. Given that Y denotes the number of days in the year that at least 700 students leave before 15:30, find
 - (i) $E(Y)$;
 - (ii) $P(Y > 150)$. [4 marks]



Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

13. [Maximum mark: 13]

(a) Given that $A = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$, show that $A^2 = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$. [3 marks]

(b) Prove by induction that

$$A^n = \begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix}, \text{ for all } n \in \mathbb{Z}^+. \quad [7 \text{ marks}]$$

(c) Given that A^{-1} is the inverse of matrix A , show that the result in part (b) is true where $n = -1$. [3 marks]

14. [Maximum mark: 16]

An open glass is created by rotating the curve $y = x^2$, defined in the domain $x \in [0, 10]$, 2π radians about the y -axis. Units on the coordinate axes are defined to be in centimetres.

(a) When the glass contains water to a height h cm, find the volume V of water in terms of h . [3 marks]

(b) If the water in the glass evaporates at the rate of 3 cm^3 per hour for each cm^2 of exposed surface area of the water, show that,

$$\frac{dV}{dt} = -3\sqrt{2\pi V}, \text{ where } t \text{ is measured in hours.} \quad [6 \text{ marks}]$$

(c) If the glass is filled completely, how long will it take for all the water to evaporate? [7 marks]

